#1.

If columns of X are not linearly independent, there exists at least one column Xi that can be represented by a linear combination of other columns. Regressing Xi on the rest columns using e=Y-Xb will result in e=0 because we already know everything about the linearly dependent Xi so we will get a perfect fit. Recall e=SSE=RSS (residual sum of squares). So SSE=RSS=0. R^2=SSR (regression sum of squares)/TSS (total sum of squares) but we also have SSE+SSR=TSS thus we can rewrite R^2=1-(SSE/TSS)=1-0=1. Thus by regressing the ith column, we can tell if linear independency exists between the columns of X.

#2.

The statement "if the covariates are linearly independent. they are uncorrelated" is false. By definition, correlation of covariates implies dependency of the variables, but not linearly dependency. An easy counter example would be biviriate normal distribution. In this case, even though X1 and X2 are linearly independent to each other, the correlation depends on Rho. If the value of Rho is non-zero, the covariates can still be correlated.

#3.1

Text

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#3.2

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#3.3

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